Thermophysical and transport properties of humid air at temperature range between 0 and 100 °C

P.T. Tsilingiris *

Department of Energy Engineering, Technological Education Institution (TEI) of Athens, A. Spyridonos Street, GR 122 10 Egaleo, Athens, Greece

Received 4 February 2007; accepted 29 September 2007
Available online 28 November 2007

Abstract

The aim of the present investigation is evaluation of the thermophysical and transport properties of moist air as a function of mixture temperature with relative humidity as a parameter, ranging between dry air and saturation conditions. Based on a literature review of the most widely available analytical procedures and methods, a number of developed correlations are presented, which are employed with recent gas mixture component properties as input parameters, to derive the temperature and humidity dependence of mixture density, viscosity, specific heat capacity, thermal conductivity, thermal diffusivity and Prandtl number under conditions corresponding to the total barometric pressure of 101.3 kPa. The derived results at an accuracy level suitable for engineering calculations were plotted and compared with adequate accuracy with existing results from previous analytical calculations and measured data from earlier experimental investigations. The saturated mixture properties were also appropriately fitted, and the fitting expressions suitable for computer calculations are also presented.

© 2007 Elsevier Ltd. All rights reserved.

Keywords: Thermophysical properties; Density; Viscosity; Specific heat capacity; Thermal conductivity; Thermal diffusivity; Prandtl number

1. Introduction

Although a considerable amount of effort has been devoted during the last several decades towards evaluation of the thermophysical and transport properties of dry air and water vapor for a wide range of temperatures, a relatively limited attention was oriented toward investigation of the corresponding properties of humid air. The development of methods for evaluation of humid air properties was the subject of a number of earlier investigations, which were employed to conduct property evaluation calculations at specific temperature regions of interest in a certain range of scientific and technological applications, like metrology and calibration as well as for air conditioning. These scientific fields of application and the corresponding investigations mainly refer either to low temperatures like those carried out by Giacomo [1], Davies [2], Zuckerwar and Meredith [3], Rasmussen [4], Hyland and Wexler [5] or to relatively higher temperatures as those by Melling et al. [6], who investigated moist air properties in the temperature range between 100 and 200 °C.

However, the knowledge of thermophysical and transport properties at intermediate temperature levels up to 100 °C is vital for certain other technological fields, like drying and water distillation, to allow accurate prediction of heat and mass transfer phenomena during the physical processes involved. With the exception of a brief report from a survey on material properties in SI units by Nelson [7], investigations on the development of correlations for derivation of the transport properties of humid air for such calculations are lacking from the literature.

When appropriate moist air data are not readily available, it is common practice to involve dry, instead of moist air properties in the associated heat transfer calculations. However, the dry air assumption cannot always be tolerated, since the use of dry air instead of moist air properties may possibly lead to considerable errors in predicting heat...
and mass transport fluxes owing to the relatively low molar mass of water vapor. This is particularly valid, especially at elevated temperatures and close to saturation conditions, when substantial amounts of water vapor are present in the mixture.

A comprehensive review of the available literature has shown an almost total lack of bibliographical reports referring to a complete account of moist air thermophysical properties as a function of relative humidity under atmospheric pressure conditions for the temperature range of interest. Although a relatively limited number of theoretical analyses and developed correlations for humid air transport properties sporadically appeared in the literature, the available data from experimental measurements are very scarce and incomplete, referring to certain fixed specific temperature levels, based almost entirely on earlier investigations.

The aim of the present investigation is to derive a full account of the commonly employed thermophysical properties of humid air at the temperature range between 0 and 100 °C as a function of the mixture degree of saturation and temperature for ordinary heat transfer engineering calculations. For this purpose, the present analysis was based on the corresponding thermophysical properties of dry air and water vapor from contemporary well established literature sources, as well as on the widely acceptable theoretical procedures in the existing literature. Along with the derivation of appropriate analytical formulations, the results corresponding to saturated mixture conditions are also presented in the form of appropriately developed polynomial fitting expressions suitable for computerized calculations. The derived results from the present investigation were also compared with corresponding data derived from existing mathematical correlations and with measured data from earlier investigations.

2. The evaluation of thermophysical and transport properties of humid air

For the purpose of determining its thermophysical and transport properties, humid air is regarded as a binary mixture of dry air and water vapor. The molar fraction of water vapor is defined as the ratio of water vapor moles to the total number of moles of the mixture as,

$$x_v = \frac{n_v}{n_m} = \frac{n_v}{n_a + n_v} = \frac{P_v}{P_0}$$  \hspace{1cm} (1)

The relative humidity is defined as,

$$\text{RH} = \frac{n_v}{n_{sv}} = \frac{x_v}{x_{sv}} = \frac{P_v}{P_{sv}}$$  \hspace{1cm} (2)

from which,
where \( x_{sv} \) is the molar fraction of the saturated water vapor under the assumption of ideal gas behavior. However, owing to interaction effects between real gas molecules, a small increase of the saturation vapor pressure in air is developed, which is taken into account by the introduction of an appropriate corrective numerical factor, known as an enhancement factor \([1,8]\). This, being a function of pressure and temperature, is introduced to correct the molar fraction of saturated vapor pressure,

\[
x_{sv} = f(P,T) \cdot \frac{P_{sv}}{P_0}
\]

(4)

The molar fraction of water vapor is then calculated from Eqs. (3) and (4) as a function of the total atmospheric pressure \( P_0 \) and the saturated vapor pressure \( P_{sv} \) at a specific temperature by the following expression,

\[
x_v = f(P,T) \cdot \text{RH} \cdot \frac{P_{sv}}{P_0}
\]

(5)

The evaluation of the numerical value of the enhancement factor was the subject of various earlier investigations, like those by Hyland and Wexler \([9] \) and Hyland \([10] \). However, for the purpose of the present analysis, its calculation was performed according to Hardy \([11] \) by the following simplified fitting expression recommended by Greenspan \([12] \)

\[
f(P,T) = \exp \left[ \xi_1 \cdot \left( 1 - \frac{P_{sv}}{P_0} \right) + \xi_2 \cdot \left( \frac{P_{sv}}{P_0} - 1 \right) \right]
\]

(6)

with

\[
\xi_1 = \sum_{i=0}^{3} A_i \cdot T^i
\]

(7)

\[
\xi_2 = \exp \left[ \sum_{i=0}^{3} B_i \cdot T^i \right]
\]

(8)

The numerical values of the constants in Eqs. (7) and (8) corresponding to the temperature range between 0 and 100 °C are \( A_0 = 3.53624 \times 10^{-6}, \) \( A_1 = 2.93228 \times 10^{-5}, \) \( A_2 = 2.61474 \times 10^{-7}, \) \( A_3 = 8.57538 \times 10^{-9}, \) \( B_0 = -1.07588 \times 10^1, \) \( B_1 = 6.32529 \times 10^{-2}, \) \( B_2 = -2.53591 \times 10^{-4} \) and \( B_3 = 6.33784 \times 10^{-7}. \) Typical calculated values of the enhancement factor as a function of temperature, which are very close to unity, are shown in Table 1. Although these numerical values appear to be lower than those derived from Giucioso \([1] \) corresponding to an appreciably lower temperature range between 0 and 30 °C, they are very slightly higher than unity, so the unity value assumption of the enhancement factor \( f = 1 \) leads to less than 0.5% maximum error at temperatures around 75 °C, which appears to be quite acceptable for the purpose of ordinary calculations. Although this assumption for the conditions corresponding to the present investigation leads to insignificant errors for ordinary engineering calculations, for any other conditions, its validity should always be properly justified.

Extensive investigations have been also conducted during the last several decades on the saturation vapor pressure of water, as reported by Alduchov and Eskridge \([13] \), mainly from the climatological and atmospheric research communities. However, these are mainly restricted to a temperature range between sub-freezing up to 40 °C, which is irrelevant to the temperature range of interest. It was, therefore, decided, for the purpose of the present investigation, to derive values of \( P_{sv} \) by the following fourth degree polynomial, which was developed by fitting the saturation vapor pressure data between 0 and 100 °C directly from the thermodynamic properties of water \([14] \) according to the following expression,

\[
P_{sv} = E_0 + E_1 \cdot t + E_2 \cdot t^2 + E_3 \cdot t^3 + E_4 \cdot t^4
\]

(9)

where \( P_{sv} \) is in kPa, for the following values of numerical constants \( E_0 = 0.7073034146, \) \( E_1 = -2.703615165 \times 10^{-2}, \) \( E_2 = 4.36088211 \times 10^{-3}, \) \( E_3 = -4.662575642 \times 10^{-5} \) and \( E_4 = 1.034693708 \times 10^{-6}. \) The previous polynomial fit expression typically offers better than 1.5% accuracy for 25 °C, which improves to about 0.2% for temperatures up to 100 °C.

2.1. Density

The evaluation of moist air density was the subject of several previous investigations covering various scientific fields of applications (1, 2 and 5, 6). The density of the binary mixture of pure water vapor and dry air at the corresponding partial pressures and molar fractions of \( P_{sv}, x_v \) and \( P_0 = P_0 - P_{sv}, x_0 = 1 - x_v, \) respectively, is calculated with sufficient accuracy through the gas equation of state, by the following simple mixing correlation,

\[
\rho_m = \frac{1}{z_m(x_v, T)} \cdot \frac{P_0}{R \cdot T} \cdot \left( M_a \cdot \frac{P_0 - P_{sv}}{P_0} + M_s \frac{P_{sv}}{P_0} \right)
\]

(10)

where \( z_m(x_v, T) \) is the compressibility factor for the gas mixture. From the above expression, the density can be derived as a function of the molar fraction of water vapor as

\[
\rho_m = \frac{1}{z_m(x_v, T)} \cdot \frac{P_0}{R \cdot T} \cdot M_a \cdot \left[ 1 - x_v \cdot \left( 1 - \frac{M_s}{M_a} \right) \right]
\]

(11)

which, combined with Eq. (5), leads to the following expression for the density of the binary mixture,

| Table 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| T (°C) | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| \( f \) | 1.0006 | 1.0010 | 1.0016 | 1.0023 | 1.0031 | 1.0039 | 1.0046 | 1.0046 | 1.0034 | 1.0000 |
| \( z_v \) | 0.9991 | 0.9987 | 0.9981 | 0.9972 | 0.9961 | 0.9947 | 0.9928 | 0.9906 | 0.9878 | 0.9844 |
\[
\rho_m = \frac{1}{z_m(x_v, T)} \cdot \frac{P_0}{R \cdot T} \cdot M_s \\
\cdot \left[ 1 - f(P, T) \cdot \frac{\rho_v}{\rho_s} \cdot \left( 1 - \frac{M_v}{M_a} \right) \cdot \left( \frac{P_{sv}}{P_0} \right) \right]
\]  

(12)

Although the compressibility factor for dry air at ambient conditions is unity, Melling et al. [6] derived its value for water vapor as a root mean square fit of data available from the literature and calculated the compressibility factor of humid air between 100 and 200 °C by the following approximate mixing expression,

\[
z_m(x_v, T) = 1 + x_v \cdot \left( \frac{a + c \cdot T}{1 + b \cdot T} - 1 \right)
\]  

(13)

with the numerical values of the constants \(a, b\) and \(c\) appropriately selected for the specified temperature range of interest. Although the calculated values of \(z_m\) from Eq. (13) were found to be close to unity, ranging between 0.9848 and 0.9947, the compressibility factor for the purpose of the present investigation, referring to the temperature region between 0 and 100 °C, was evaluated from the virial equation of state according to the following expression,

\[
z_v = 1 + A \cdot P_{sv} + B \cdot P_{sv}^2
\]  

(14)

which was recommended by Hyland and Wexler [8]. The second and third pressure series virial coefficients were calculated by,

\[
A = C_1 + C_2 \cdot e^C_3
\]  

(15)

\[
B = K_1 + K_2 \cdot e^K_3
\]  

(16)

with \(C_1 = 0.7 \times 10^{-8} \text{ Pa}^{-1}\), \(C_2 = -0.147184 \times 10^{-8} \text{ Pa}^{-1}\), \(C_3 = 1734.29 \text{ (K)}^{-1}\), \(K_1 = 0.104 \times 10^{-14} \text{ Pa}^{-2}\), \(K_2 = -0.335297 \times 10^{-12} \text{ Pa}^{-2}\) and \(K_3 = 3645.09 \text{ K}^{-1}\). Typical calculated values of \(z_v\), which were found to be very close to unity, are shown in Table 1 as a function of temperature. Fixing the compressibility factor to the unity value \(z_v = 1\) leads to very small errors for the calculation of mixture density, typically about 0.38% for 50 °C and less than about 1.5% for 100 °C. The combined effect of fixing both the enhancement and compressibility factors at the unity value is estimated to be responsible for an overall maximum error for the evaluation of density that ranges between about 0.4% and less than 1.5% at the corresponding temperatures of 0 and 100 °C. Although the assumption of a unity value for the compressibility and enhancement factors leads to insignificant errors for the purpose of the present analysis, its validity should always be properly justified for specific conditions, when sacrifice of improved accuracy cannot be tolerated.

2.2. Viscosity

Based on further kinetic theory approach considerations, Reid et al. [15] recommended the following expression for the viscosity of a mixture of dilute gases with \(i\) components, which was based on earlier investigations by Wilke [16],

\[
\mu_m = \sum_{i=1}^{n} \frac{x_i \cdot \mu_i}{\sum_{j=1}^{n} x_j \Phi_{ij}}
\]  

(17)

with the interaction parameters \(\Phi_{ij}\) and \(\Phi_{ji}\) given by,

\[
\Phi_{ij} = \frac{1 + \left( \frac{\mu_i}{\mu_j} \right)^{1/2} \cdot \left( \frac{M_j}{M_i} \right)^{1/4} \cdot \left( \frac{P_{sv}}{P_0} \right)^{2/3}}{8 \left[ 1 + \frac{M_j}{M_i} \right]^{1/2}}
\]  

(18)

\[
\Phi_{ji} = \frac{\mu_j}{\mu_i} \cdot \frac{M_j}{M_i} \cdot \Phi_{ij}
\]  

(19)

As derived from Eq. (18), \(\Phi_{ij} = \Phi_{ji} = 1\), which, combined with Eq. (17), after its expansion, leads to the following expression,

\[
\mu_m = \frac{(1 - x_v) \cdot \mu_a}{(1 - x_v) + x_v \cdot \Phi_{av}} + \frac{x_v \cdot \mu_v}{x_v + (1 - x_v) \cdot \Phi_{va}}
\]  

(20)

Taking into consideration Eq. (5), the previous expression becomes,

\[
\mu_m = \frac{1 - f(P, T) \cdot \rho_v}{1 - f(P, T) \cdot \rho_v \cdot \Phi_{av} + f(P, T) \cdot \rho_v \cdot \Phi_{va}} \cdot \frac{1 - f(P, T) \cdot \rho_v}{1 - f(P, T) \cdot \rho_v \cdot \Phi_{av} + f(P, T) \cdot \rho_v \cdot \Phi_{va}} \cdot \mu_a
\]  

\[
+ f(P, T) \cdot \rho_v \cdot \Phi_{va} + \left[ 1 - f(P, T) \cdot \rho_v \cdot \Phi_{av} + f(P, T) \cdot \rho_v \cdot \Phi_{va} \right] \cdot \Phi_{va}
\]  

(21)

which offers the viscosity of a humid air mixture at a specific temperature and relative humidity as a function of dry air and water vapor viscosities for the following values of the interaction parameters,

\[
\Phi_{av} = \frac{\sqrt{2}}{4} \cdot \left( 1 + \frac{M_v}{M_a} \right)^{-1/4} \cdot \left[ 1 + \frac{\mu_v}{\mu_a} \cdot \left( \frac{M_v}{M_a} \right)^{1/4} \right]^{1/2}
\]  

(22)

\[
\Phi_{va} = \frac{\sqrt{2}}{4} \cdot \left( 1 + \frac{M_v}{M_a} \right)^{-1/4} \cdot \left[ 1 + \frac{\mu_v}{\mu_a} \cdot \left( \frac{M_v}{M_a} \right)^{1/4} \right]^{1/2}
\]  

(23)

2.3. Thermal conductivity

Reid et al. [15] suggests the following expression, which was originally proposed by Wassiljewa [17], as the basis of the calculation of the thermal conductivity of the mixture,

\[
k_m = \sum_{i=1}^{n} \frac{x_i \cdot k_i}{\sum_{j=1}^{n} x_j \cdot \Phi_{ij}}
\]  

(24)

Based also on the original investigations by Mason and Saxena [18], they have recommended that,
where the ratio of the monoatomic values of thermal conductivity in the previous expression is calculated according to Ref. [15] by,

$$\Theta_{ij} = e \cdot \left[ 1 + \left( \frac{\kappa_i}{\kappa_m} \right)^{1/2} \cdot \left( \frac{M_i}{M_m} \right)^{1/4} \right]^2$$  (25)

Mason and Saxena [18] suggested that although \(e\) can be a complex function of several kinetic parameters, its numerical value, which is never far from unity, can be set equal to some best average value, which they recommended to be \(e = 1.065\) for non-polar gases. Tondon and Saxena [19], suggested that better accuracy can be obtained by fixing this numerical value to \(e = 0.85\) for a mixture of polar and non-polar gases, while according to Reid et al. [15], this numerical constant was recommended to be fixed at its unity numerical value \(e = 1\), which was also adopted here for the purpose of the subsequent calculations.

Under this assumption, the substitution of Eq. (26) into Eq. (25) leads to,

$$\phi_{ij} = \Theta_{ij}$$  (27)

The following expression, employed for calculation of the thermal conductivity of mixtures, is derived from Eq. (24),

$$k_m = \frac{(1 - x_v) \cdot k_a + x_v \cdot k_v}{x_v + (1 - x_v) \cdot \phi_{av}}$$  (28)

Taking into consideration Eq. (5), the previous expression becomes,

$$k_m = \frac{\left[ 1 - f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) \right] \cdot k_a}{1 - f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) + f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) \cdot \phi_{av}} + \frac{f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) \cdot k_v}{f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) + 1 - f(P, T) \cdot \text{RH} \cdot \left( \frac{\rho_m}{\rho_0} \right) \cdot \phi_{av}}$$  (29)

This expression is very similar to the corresponding Eq. (21) for calculation of the mixture viscosity, and it is employed for evaluation of the mixture thermal conductivity as a function of the corresponding values of the humid air components.

2.4. Specific heat capacity

The general procedure for evaluating the specific heat capacity is to apply a simple linear mixing equation as proposed by Wong and Embelton [20], who derived the heat capacity of humid air as a function of relative humidity for the limited temperature range between 0 and 30 °C. The same procedure was also adopted by several other investigators, like Zuckerwar and Meredith [3], Durst et al. [21], although for a non-clearly specified temperature range, and Rasmussen [4], who derived correlations for a relatively narrow temperature range around ambient temperatures, as well as Melling et al. [6], who reported data between 100 and 200 °C. Following the same approach, the specific heat capacity of the ideal gas mixture can be expressed as

$$c_{pm} = c_{pa} \cdot x_a \cdot \frac{M_a}{M_m} + c_{pv} \cdot x_v \cdot \frac{M_v}{M_m}$$  (30)

To account for the real gas behavior, the correction term \(\Delta c_p\) was proposed according to Reid et al. [15].

$$c_{pm} - c_{p,m} = \Delta c_p$$  (31)

which is a complex function of the first- and second-order deviation functions and theacentric factor of the mixture molecules. This factor represents the influence ofacentricity or non-sphericity of the constituent gas molecules, which, although for monoatomic gases it is zero, increases with molecular weight and molecular structure complexity with respect to both geometry and polarity. The numerical value of the correction term \(\Delta c_p\) was found, according to Melling et al. [6] for the specific conditions, to be comparatively small, leading to the maximum 1.5% corrections to ideal gas behavior, and therefore, its effect was assumed to be negligible, leading to \(c_{p,m} = c_{pm}\) for the purpose of the subsequent calculations.

However, taking into account Eq. (5), the molar fraction of dry air is

$$x_a = 1 - x_v = 1 - f(P, T) \cdot \frac{\text{RH} \cdot \frac{\rho_m}{\rho_0}}{P_0}$$  (32)

and since

$$M_m = M_a \cdot x_a + M_v \cdot x_v$$  (33)

Eq. (30) becomes,

$$c_{pm} = \frac{c_{pa} \cdot \left( 1 - f(P, T) \cdot \text{RH} \cdot \frac{\rho_m}{\rho_0} \right) \cdot M_a + c_{pv} \cdot f(P, T) \cdot \text{RH} \cdot \frac{\rho_m}{\rho_0} \cdot M_v}{M_a \cdot \left( 1 - f(P, T) \cdot \text{RH} \cdot \frac{\rho_m}{\rho_0} \right) + M_v \cdot f(P, T) \cdot \text{RH} \cdot \frac{\rho_m}{\rho_0}}$$  (34)

2.5. Thermal diffusivity

Thermal diffusivity is calculated from its definition expression,

$$a_m = \frac{k_m}{\rho_m \cdot c_{pm}}$$  (35)

taking into account the previously derived Eqs. (12), (29) and (34) for the respective calculations of density, thermal conductivity and specific heat capacity of humid air.

2.6. Prandtl number

This dimensionless number is defined as \(Pr_m = \nu_m / a_m\). Since the thermal diffusivity from Eq. (35) and the kine-
matic viscosity, as derived from \( \mu_m = \rho_m \cdot v_m \), are both functions of density, the Prandtl number is evaluated as a function of known thermophysical properties \( \mu_m \), \( c_p m \) and \( k_m \), as

\[
P_r m = \frac{\mu_m \cdot c_p m}{k_m}
\]

with the values of properties \( \mu_m \), \( c_p m \) and \( k_m \) derived from Eqs. (21), (34) and (29), respectively.

3. The thermophysical and transport properties of dry air and water vapor

Dry air is a mixture of several gas components at different concentrations, with a composition that can approximately be considered to be constant in the atmosphere. As soon as the molar mass \( M_{ai} \) and the molar fraction \( x_{ai} \) of the \( n \) individual constituent gases of dry air mixture are known, the molar mass of atmospheric air can be derived by the following expression,

\[
M_a = \frac{\sum_{i=1}^{n} x_{ai} \cdot M_{ai}}{\sum_{i=1}^{n} x_{ai}}
\]

(37)

Assuming a composition similar to that of standard dry air, as described for example by Ref. [22], a molar mass \( M_a = 28.9635 \) kg/kmol is derived from the previous expression.

The calculation of the thermodynamic and transport properties of dry air and water vapor mixtures is based on the existing properties of the constituent gases, both of which have been the objective of extensive research during the last several decades. Since the selection of these properties as input data for the subsequent calculations is crucial for accurate evaluation of the corresponding humid air properties, one has to rely on recent, accurate and reliable data from one among the several available, well-established literature sources. From this point of view, all selected properties were derived from the Handbook of Heat Transfer [23], in which data for the viscosity, thermal conductivity and specific heat capacity of dry air are available, as compiled from Irvine and Liley [24].

The viscosity of dry air in Ns/m² \( \times 10^{-6} \) is offered by the following correlation,

\[
\mu_a = M_{a0} + M_{a1} \cdot T + M_{a2} \cdot T^2 + M_{a3} \cdot T^3 + M_{a4} \cdot T^4
\]

(38)

in the temperature range \(-23^\circ C \leq t \leq 327^\circ C\), for the following values of the numerical constants, \( M_{a0} = -9.8601 \times 10^{-1} \), \( M_{a1} = 9.080125 \times 10^{-2} \), \( M_{a2} = -1.17635575 \times 10^{-4} \), \( M_{a3} = 1.2349703 \times 10^{-7} \) and \( M_{a4} = -5.7971299 \times 10^{-11} \).

The thermal conductivity of dry air in W/m K \( \times 10^{-3} \) is expressed by the following correlation,

\[
k_a = K_{a0} + K_{a1} \cdot T + K_{a2} \cdot T^2 + K_{a3} \cdot T^3 + K_{a4} \cdot T^4
\]

(39)

for \( K_{a0} = -2.276501 \times 10^{-3} \), \( K_{a1} = 1.2598485 \times 10^{-4} \), \( K_{a2} = -1.4815235 \times 10^{-7} \), \( K_{a3} = 1.73550646 \times 10^{-10} \), \( K_{a4} = -1.066657 \times 10^{-13} \) and \( K_{a5} = 2.47663055 \times 10^{-17} \). This expression is valid at the temperature range between \(-23^\circ C \leq t \leq 777^\circ C\).

The specific heat capacity of dry air in kJ/kg K, which is valid for the same temperature range, is given by the following expression

\[
c_{pa} = C_{a0} + C_{a1} \cdot T + C_{a2} \cdot T^2 + C_{a3} \cdot T^3 + C_{a4} \cdot T^4
\]

(40)

for the following values of the numerical constants \( C_{a0} = 0.103409 \times 10^1 \), \( C_{a1} = -0.284887 \times 10^{-2} \), \( C_{a2} = 0.7816818 \times 10^{-6} \), \( C_{a3} = -0.4970786 \times 10^{-9} \) and \( C_{a4} = 0.1077024 \times 10^{-12} \).

Corresponding data for the water vapor properties were taken from the same literature source as compiled from Toulukian et al. [25]. These data were fitted by the following appropriate degree polynomials in the temperature range \( 0 \leq t \leq 120^\circ C\).

The viscosity in Ns/m² \( \times 10^{-6} \) was determined by the following linear expression,

\[
\mu_v = M_{v0} + M_{v1} \cdot t
\]

(41)

with \( M_{v0} = 8.058131868 \times 10^1 \) and \( M_{v1} = 4.000549451 \times 10^{-1} \).

The thermal conductivity in W/m K \( \times 10^{-3} \) was determined by the expression,

\[
k_v = K_{v0} + K_{v1} \cdot t + K_{v2} \cdot t^2
\]

(42)

where \( K_{v0} = 1.761758242 \times 10^1 \), \( K_{v1} = 5.558941059 \times 10^{-2} \) and \( K_{v2} = 1.663336663 \times 10^{-4} \).

The specific heat capacity in (kJ/kg K) was determined by the following expression,

\[
c_{pv} = C_{v0} + C_{v1} \cdot t + C_{v2} \cdot t^2
\]

(43)

with \( C_{v0} = 1.86910989 \), \( C_{v1} = -2.578421578 \times 10^{-4} \) and \( C_{v2} = 1.941058941 \times 10^{-5} \).

4. Results and discussion

The derived density from Eq. (12) is plotted in Fig. 1 as a function of temperature for an increasing relative humidity from the minimum value of 0%, corresponding to dry air (top curve), up to the maximum value of RH = 100% (lower curve), corresponding to saturated conditions, in 10% steps. The increase of relative humidity leads to a decrease of humid air density, especially at the range of higher temperatures, owing to \( M_a - M_d > 0 \). Although a relative humidity increase between 0 and 100% leads to an almost negligible density decrease of humid air at near freezing temperatures, it is responsible for a density decrease of about 4.8% at 50°C, and to a significant reduction of humid air density of about 37.5%, at temperatures close to 100°C.
The density of the saturated mixture for the temperature range between 0 and 100 °C was fitted by the following third degree polynomial,

\[ \rho_m = SD_0 + SD_1 \cdot t + SD_2 \cdot t^2 + SD_3 \cdot t^3 \] (44)

with a coefficient of determination and the values of the numerical constants SD_0 to SD_3 as shown in Table 4.

The humid air viscosity is plotted in Fig. 2 as a function of temperature for parametric values of relative humidity ranging between 0% and 100% in 10% steps.

The thermal conductivity of humid air as derived from Eq. (29) is plotted in Fig. 3 for the parametric values of relative humidity ranging between 0% (top curve) and 100% (lower curve) in fixed 10% relative humidity steps. A similar effect of the relative humidity increase between 0% and 100% is demonstrated as before, which, although at the lower temperatures leads to a negligible decrease, at temperatures around 50 °C leads to a decrease of 3.5% and...
at temperatures around 100 °C to a significant decrease of about 21.5% in thermal conductivity.

A maximum value of thermal conductivity is developed for each fixed relative humidity curve, which moves towards higher temperatures as the relative humidity decreases. This maximum moves typically from the temperature of 63 °C to about 94 °C as the relative humidity decreases from saturation level conditions to about RH = 40%.

The thermal conductivity of the saturated mixture for the temperature range of interest was fitted by the following fourth degree polynomial,

\[ k_m = S_0 t^0 + S_1 t^1 + S_2 t^2 + S_3 t^3 + S_4 t^4 \]

with a coefficient of determination and the values of the numerical constants S0–S4 as shown in Table 4.

The specific heat capacity of humid air as derived from Eq. (34) is plotted in Fig. 4 with the relative humidity as a parameter, increasing from the value of RH = 0% (lower line) to 100% (top line) in 10% steps. Again, although the increase of relative humidity up to its saturation level leads to an almost negligible increase of viscosity at temperatures close to 0 °C, its effect leads to an increase of about 7% and 100% at the corresponding temperatures of 50 and 100 °C, respectively. The saturated mixture specific heat capacity for the temperature range between 0 and 100 °C was fitted by the following fifth degree polynomial,

\[ c_{pm} = S_0 t^0 + S_1 t^1 + S_2 t^2 + S_3 t^3 + S_4 t^4 + S_5 t^5 \]

with the corresponding coefficient of determination and values of the numerical constants S0 to S5 shown in Table 4.

The thermal diffusivity of humid air is plotted in Fig. 5 as derived from Eq. (35) for an increasing parametric value of relative humidity between RH = 0% (top line) to 100% (lower line) in uniform 10% steps. The corresponding saturated mixture properties for the temperature range of interest were fitted by the following fourth degree polynomial,

\[ \alpha_m = S_0 t^0 + S_1 t^1 + S_2 t^2 + S_3 t^3 + S_4 t^4 \]

with a coefficient of determination and the values of the numerical constants S0 to S4 as shown in Table 4.
Finally, the temperature dependence of the moist air Prandtl number for a relative humidity ranging between RH = 0% (lower curve) to 100% (top curve) as a parameter is shown in Fig. 6.

A similar behavior of the growing dependence of both properties, the thermal diffusivity and Prandtl number with the relative humidity, especially at elevated mixture temperatures is exhibited as shown in Figs. 5 and 6. The Prandtl number for the saturated mixture for the same temperature range was fitted by the following fourth degree polynomial,

\[ Pr_m = SP_0 + SP_1 \cdot t + SP_2 \cdot t^2 + SP_3 \cdot t^3 + SP_4 \cdot t^4 \]

(49)

with a coefficient of determination and corresponding numerical constants as shown in Table 4.

5. Comparisons with the results from earlier investigations

Nelsons report [7] with the proposed empirical correlations appears to be the single readily available source of formulae suitable for the derivation of moist air thermophysical properties to fit data for the temperature range of interest. However, references or citations on the origin of the recommended expressions, except for Ref. [26], are missing and detailed derivations are completely lacking from his original paper.

The recommended expression for mixture density is,

\[ \rho_m = (3.484 - 1.317 \cdot x_s) \cdot \frac{P_0}{273.15 + t} \]

(50)

As derived from a careful inspection, the previous expression can easily be derived directly from Eq. (12), taking into consideration Eq. (5) and assuming unity values for \( z_m \) and \( f(P, T) \), as well as the standard numerical values of the following constants \( R = 8.314 \text{ J/mol K} \), \( M_a = 28.963 \text{ kg/kmol} \) and \( M_v = 18.02 \text{ kg/kmol} \), leading in this way to identical results with those of Eq. (12).

The proposed expression for mixture viscosity is,

\[ \mu_m = \frac{\mu_a + x_v \cdot (0.7887 \cdot \mu_v - \mu_a)}{1 - 0.2113 \cdot x_v} \]

(51)

for which the report referred to Ref. [26], where its derivation is attributed to an earlier cited work by Herwig and Zipperer recommending the derivation of mixture viscosity as,

\[ \mu_m = \frac{\sum_{i=1}^{n} x_i \cdot \mu_i \cdot (M_i)^{1/2}}{\sum_{i=1}^{n} x_i \cdot (M_i)^{1/2}} \]

(52)

The above expression, applied to a two component system, namely dry air and water vapor, is identical to Eq. (9) in Ref. [6], as proposed by Krischer and Kast [27], for the same molecular weights of mixture components. This expression is completely different from Eq. (21), which was employed for the purpose of the present investigation.

The proposed expression for the specific heat capacity, although of an unspecified origin is,

\[ c_{pm} = c_{pa} + 0.622 \cdot \frac{P_v}{P_0 - P_v} \]

(53)

It can easily be seen that this expression can be derived from Eq. (34) based on linear mixing considerations by omitting the water vapor pressure contribution quantity \( M_v \cdot f(P, T) \cdot RH \cdot (P_{m}/P_0) \) of the denominator and assuming a unity value for the water vapor specific heat capacity, which, however, for the temperature region of interest, ranges between about 1.86-2.03 kJ/kg K. Owing to the fact that these assumptions are not valid, Eq. (53) appears to be of questionable accuracy.

The proposed thermal conductivity expression in (mW/m K) units is,

\[ k_m = \frac{k_a + x_v \cdot (0.8536 \cdot k_v - k_a)}{1 - 0.1464 \cdot x_v} \]

(54)

This is also attributed to the chemical engineers handbook [26] in which its derivation is attributed to the following expression, recommended by Friend and Adler [28],

\[ k_m = \frac{\sum_{i=1}^{n} x_i \cdot k_i \cdot (M_i)^{1/3}}{\sum_{i=1}^{n} x_i \cdot (M_i)^{1/3}} \]

\[ = \frac{x_v \cdot k_v \cdot (M_v)^{1/3} + x_s \cdot \mu_v \cdot (M_v)^{1/3}}{x_v \cdot (M_a)^{1/3} + x_s \cdot (M_v)^{1/3}} \]

(55)

from which Eq. (54) is derived for the numerical constants of \( M_a = 28.963 \text{ kg/kmol} \) and \( M_v = 18.02 \text{ kg/kmol} \), which is completely different than the derived Eq. (29).

The existing experimental measurements of mixture thermophysical properties in the literature are scarce, based mostly on earlier investigations conducted at certain specific temperature levels for a relatively wide range of water vapor molar fraction. Since the current investigation covers mixture properties at a constant pressure corresponding to normal atmospheric conditions, the comparative presentation of the derived results and existing measured data requires the reference of the water vapor molar fraction of the measurements to the corresponding relative humidity of the mixture up to its saturation level. This was calculated from the corresponding molar fraction and the saturated vapor pressure at the specified temperature based on Eq. (5) for a unity value of the enhancement factor.

The existing viscosity measurements, as compiled from the literature, are shown in Table 2 with the corresponding approximate evaluated relative humidity levels rounded to their nearest integer value. The measured data from Kestin and Whitelaw, Hochrainer and Munczak and Vargaftik [29] were taken from the Tables 5 to 7 of Ref. [6], while results derived from Mason and Monchick were compiled graphically from plotted data by Kestin and Whitelaw [30]. The thermal conductivity data for the temperature level of 80°C were evaluated from measured \( k_m/k_a \) ratios originally derived by Gruss and Schmick [31] and from published data by Vargaftik [29], while for 60°C, they were taken from Touloukian et al. [32] as reported in Ref. [6].
In Fig. 7, the derived viscosity, represented by thick solid lines of increasing relative humidity in uniform 10% steps, is compared to the corresponding results, plotted with broken lines, calculated through Eq. (43) as proposed by Nelson and to various measurements from the literature. It appears that although not identical, there is a remarkable agreement between the results from the present investigation and Nelson’s empirical correlation, the maximum difference being typically less than about 4% and mainly confined to different slopes between uniform relative humidity curves. The discrete data points, representing earlier measurements for certain corresponding fixed temperatures, cover almost the entire relative humidity range, approximately up to the saturation level, in increasing although arbitrary steps, as can also be seen from Table 2. The measured data appear to be slightly higher than both the calculated mixture viscosity from the present investigation and the results from Nelson’s correlation, especially at the mid temperature range of interest, the deviations being typically lower than 8%.

The corresponding results referring to thermal conductivity are also shown in Fig. 8 in which the deviations between the results from the present investigation and Nelson’s data again appear to be small, typically about 1%, and mostly confined to slight slope differences between corresponding fixed relative humidity curves over the entire temperature range of interest. Thermal conductivity measurements in the literature appear to be very scarce.

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data corresponding to viscosity measurements from various literature sources (×10⁻⁶ kg/m s)</td>
</tr>
<tr>
<td>From Kestin and Whitelaw [6]</td>
</tr>
<tr>
<td>25 °C</td>
</tr>
<tr>
<td>19%</td>
</tr>
<tr>
<td>15.5%</td>
</tr>
<tr>
<td>14%</td>
</tr>
<tr>
<td>From Hochrainer and Munczak [6]</td>
</tr>
<tr>
<td>20 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>30 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>From Vargaftik [6,29]</td>
</tr>
<tr>
<td>50 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>60 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>70 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>80 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>From Mason and Monchick [30]</td>
</tr>
<tr>
<td>25 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>50 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>75 °C</td>
</tr>
<tr>
<td>0%</td>
</tr>
</tbody>
</table>
while most different reports, like Vargaftik [29], Toulouki-an [32] and Tondon and Saxena [19], repeatedly cite the earlier measurements by Gruss and Schmick [31], who measured the \((k_m/k_a)\) ratio at 80 °C, from which, for the purpose of the present investigation, \(k_m\) was calculated using dry air properties from Ref. [24]. Touloukian [32] also reported measurements from nitrogen/water vapor mixtures, since nitrogen is the major constituent of dry air. The available limited measurements covering the entire relative humidity range approximately up to the saturation level in arbitrary increasing steps, as can be seen in Table 3, were included as discrete data points in Fig. 7. Although these early measurements are inadequate for establishing a complete reliable set of validation data, they are indicating values higher than those predicted from the present analysis and Nelson's correlation, by a maximum level of about 7% corresponding to 65 °C and by 10% to 80 °C, which still compares sufficiently well with theory.

Comparative results for the specific heat capacity of mixtures as derived from the present analysis (group of thick solid lines) and according to the correlation Eq. (45) (group of broken lines) can be seen in Fig. 9, for relative humidity varying between 0%, (lower line in each group) and saturation conditions (upper line). Apparently, Nelson’s correlation, owing to the reasons discussed previously in detail, leads to unrealistically higher values of moist air specific heat capacity, especially at the higher relative humidity and temperature levels.

### 6. Conclusions

The present investigation has allowed the derivation of a complete account of the thermophysical and transport properties of moist air in the temperature range between 0 and 100 °C under conditions corresponding to normal barometric pressure of 101.3 kPa, which is of major importance in several fields of technology. The evaluation of properties was based on a comprehensive literature review for the appropriate selection of the most widely acceptable procedures and methods as well as dry air and water vapor thermophysical properties, which were employed as input parameters for the subsequent calculations. The methodology and the derived corresponding analytical correlations were presented and employed for calculation of the dry air and water vapor mixture density, viscosity, thermal

### Table 3

<table>
<thead>
<tr>
<th>From Gruss and Schmick [31]</th>
<th>(80^\circ\text{C})</th>
<th>2.9845</th>
<th>3.044</th>
<th>3.088</th>
<th>3.094</th>
<th>3.091</th>
<th>3.088</th>
<th>3.0949</th>
<th>3.062</th>
<th>3.074</th>
<th>2.9810</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td></td>
<td>15%</td>
<td>32%</td>
<td>37%</td>
<td>42%</td>
<td>48%</td>
<td>53%</td>
<td>65%</td>
<td>67%</td>
<td>95%</td>
<td></td>
</tr>
<tr>
<td>From Vargaftik [29]</td>
<td>(80^\circ\text{C})</td>
<td>2.989</td>
<td>3.069</td>
<td>3.103</td>
<td>3.078</td>
<td>3.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td></td>
<td>21%</td>
<td>43%</td>
<td>65%</td>
<td>86%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From Touloukian et al. [32,6]</td>
<td>(60^\circ\text{C})</td>
<td>2.92</td>
<td>2.96</td>
<td>2.96</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td></td>
<td>40%</td>
<td>81%</td>
<td>&gt;100%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4
The numerical constants and coefficients of determination (COD) for the proposed polynomial fit expressions for the following saturated mixture properties, density, viscosity, thermal conductivity, specific heat capacity, thermal diffusivity and Prandtl number:

<table>
<thead>
<tr>
<th>Property</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
<th>COD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>0.999954</td>
<td>0.999997</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity (Ns/m²)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity (W/mK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific heat capacity (kJ/kg K)</td>
<td>0.999905</td>
<td>0.999900</td>
<td>0.999990</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity (m²/s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prandtl number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD₃ = 1.293393662</td>
<td>SV₀ = 1.715747771E-5</td>
<td>SK₀ = 2.40073953E-2</td>
<td>SC₀ = 1.004571427</td>
<td>SA₀ = 1.847185729E-5</td>
<td>SP₀ = 0.7215798365</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD₂ = -5.538444326E⁻³</td>
<td>SV₁ = 4.722402075E-8</td>
<td>SK₁ = 7.278410162E-5</td>
<td>SC₁ = 2.05063275E-3</td>
<td>SA₁ = 1.161914598E-7</td>
<td>SP₁ = -3.703124976E⁻⁴</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD₁ = 3.860201577E⁻⁵</td>
<td>SV₂ = -3.663027156E⁻¹₀</td>
<td>SK₂ = -1.788037411E⁻²</td>
<td>SC₂ = -1.6351537093E⁻⁴</td>
<td>SA₂ = 2.373056947E⁻¹⁰</td>
<td>SP₂ = 2.240599044E⁻⁵</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD₀ = 1.293393662</td>
<td>SV₃ = 1.873236686E⁻¹₂</td>
<td>SK₃ = -1.351703529E⁻⁹</td>
<td>SC₃ = 6.2123003E⁻⁶</td>
<td>SA₃ = -5.769352751E⁻¹²</td>
<td>SP₃ = -4.162785412E⁻⁷</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SC₅ = 5.071307038E⁻¹⁰</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9. Comparative specific heat capacity of moist air at various temperature levels according to results from the present analysis (curves in solid lines) and Nelsons correlation (53) (curves in broken lines) for a relative humidity ranging between RH = 0% (lower curves) to RH = 100% (top curves).

conductivity, specific heat capacity, thermal diffusivity and Prandtl number data, which are necessary for performing heat transfer calculations corresponding to an appropriate accuracy level, suitable for ordinary engineering calculations. The derived results were graphically presented for the temperature range of interest with the relative humidity varying between dry and saturated conditions as a parameter. The saturated mixture properties were also fitted and the derived fitting expressions suitable for computerized calculations were also presented. The results from the present investigation were also compared with corresponding previously reported data, either analytical or measured, from the literature. The comparative presentation shows that, with the exception of the specific heat capacity, there is very good agreement between the results from the present investigation and previous analyses and good agreement with the scarce and mostly incomplete sporadically appearing results from earlier measurements in the literature.

References


